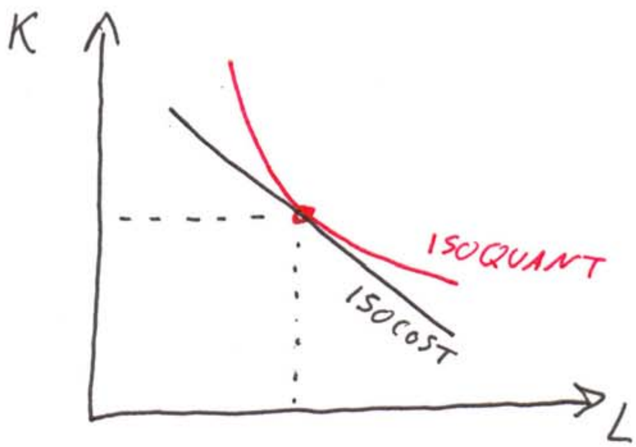
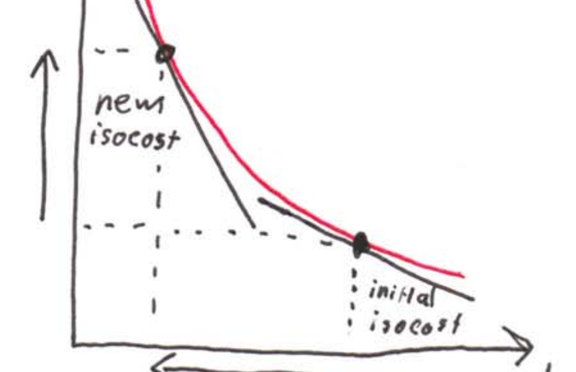
STOLPER-SAMUELSON EXAMPLE

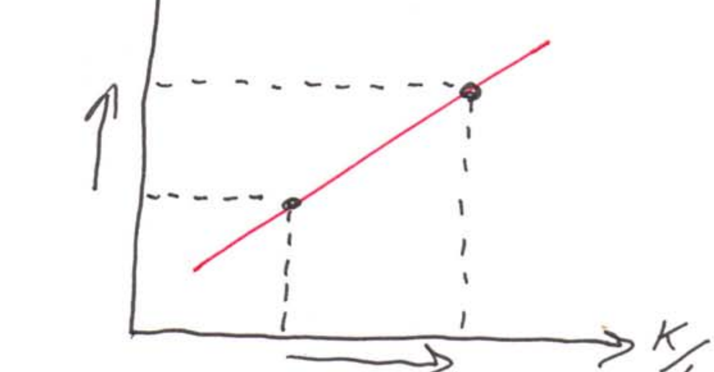
Recall that a cost minimizing firm selects the optimal amounts of capital, K and labor, L at the point where the isoquant is tangent to the isocost

$$\underset{\text{isocost}}{\text{slope of}} \quad \frac{w}{r} = \frac{MPL}{MPK} \quad \underset{\text{isoquant}}{\text{slope of}}$$



If the relative wage rises, the firm's capital-labor ratio would rise





Suppose that there are two sectors Cloth and Manufactures

Production Function in Cloth sector

$$C = K_c^{\alpha} L_c^{1-\alpha}$$

$$\frac{MPL_c}{MPK_c} = \frac{(1-\alpha) K_c^{\alpha} L_c^{-\alpha}}{\alpha K_c^{\alpha-1} L_c^{1-\alpha}}$$

$$\boxed{\frac{MPL_c}{MPK_c} = \frac{1-\alpha}{\alpha} \cdot \frac{K_c}{L_c}}$$

slope of isoquant in the cloth sector

$$\frac{w}{r} = \frac{1-\alpha}{\alpha} \cdot \frac{K_c}{L_c}$$

OR

$$\frac{K_c}{L_c} = \frac{\alpha}{1-\alpha} \cdot \frac{w}{r}$$

Production Function in Manuf. sector

$$M = K_M^{1-\alpha} L_M^{\alpha}$$

$$\frac{MPL_M}{MPK_M} = \frac{\alpha K_M^{1-\alpha} L_M^{\alpha-1}}{(1-\alpha) K_M^{-\alpha} L_M^{\alpha}}$$

$$\boxed{\frac{MPL_M}{MPK_M} = \frac{\alpha}{1-\alpha} \cdot \frac{K_M}{L_M}}$$

slope of isoquant in the manufactures sector

$$\frac{w}{r} = \frac{\alpha}{1-\alpha} \cdot \frac{K_M}{L_M}$$

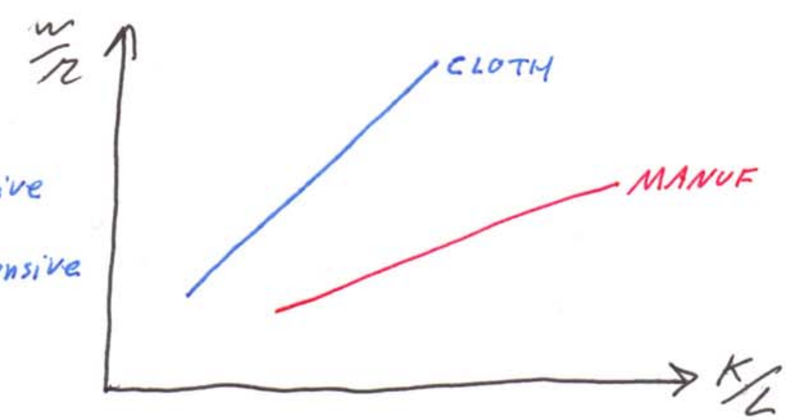
OR

$$\frac{K_M}{L_M} = \frac{1-\alpha}{\alpha} \cdot \frac{w}{r}$$

If $\alpha = 1/3$, then

cloth is labor-intensive

manuf. is capital-intensive



Next recall that at an optimum the relative price of cloth must be equal to the opportunity cost of producing cloth

$$\text{rel price of cloth} \quad \frac{P_C}{P_M} = \frac{MPL_M}{MPL_C} = \frac{MPK_M}{MPK_C} \quad \text{opp cost of producing cloth}$$

$$\frac{P_C}{P_M} = \frac{\alpha K_M^{1-\alpha} L_M^{\alpha-1}}{(1-\alpha) K_C^{\alpha} L_C^{-\alpha}} = \frac{(1-\alpha) K_M^{-\alpha} L_M^{\alpha}}{\alpha K_C^{\alpha-1} L_C^{1-\alpha}}$$

$$\frac{P_C}{P_M} = \frac{\alpha}{1-\alpha}\left(\frac{K_M}{L_M}\right)^{1-\alpha}\left(\frac{K_C}{L_C}\right)^{-\alpha} = \frac{1-\alpha}{\alpha}\left(\frac{K_M}{L_M}\right)^{-\alpha}\left(\frac{K_C}{L_C}\right)^{1-\alpha}$$

$$\left(\frac{K_M}{L_M}\right)^{\alpha}\left(\frac{K_C}{L_C}\right)^{\alpha}\frac{P_C}{P_M} = \frac{\alpha}{1-\alpha}\cdot\frac{K_M}{L_M} = \frac{1-\alpha}{\alpha}\cdot\frac{K_C}{L_C} = \frac{w}{r}$$

$$\frac{w}{r} = \frac{P_C}{P_M}\cdot\left(\frac{K_C}{L_C}\right)^{\alpha}\left(\frac{K_M}{L_M}\right)^{\alpha}$$

but $\dfrac{K_C}{L_C} = \dfrac{\alpha}{1-\alpha}\cdot\dfrac{w}{r}$ and $\dfrac{K_M}{L_M} = \dfrac{1-\alpha}{\alpha}\cdot\dfrac{w}{r}$

$$\frac{w}{r} = \frac{p_c}{p_M} \cdot \left(\frac{\alpha}{1-\alpha} \cdot \frac{w}{r}\right)^{\alpha} \left(\frac{1-\alpha}{\alpha} \cdot \frac{w}{r}\right)^{\alpha}$$

$$\frac{w}{r} = \frac{p_c}{p_M} \cdot \left(\frac{w}{r}\right)^{2\alpha} \cdot \underbrace{\left(\frac{\alpha}{1-\alpha}\right)^{\alpha} \cdot \left(\frac{1-\alpha}{\alpha}\right)^{\alpha}}_{\text{one}}$$

$$\boxed{\frac{p_c}{p_M} = \left(\frac{w}{r}\right)^{1-2\alpha}}$$

If $\alpha = 1/3$ then cloth is labor-intensive and manufactures are capital-intensive

So if $\alpha = 1/3$ then an increase in the price of the labor-intensive good (cloth) will raise the relative wage

Also note that if $\alpha = 2/3$ then cloth would be capital intensive and $1-2\alpha < 0$, so an increase in the price of cloth would reduce the relative wage.